1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for $f(x) = 2x^2 + 8x - 3$. Then graph the function by making a table of values.

SOLUTION:

Here, $a = 2$, $b = 8$, and $c = -3$.

To find the y-intercept, substitute $x = 0$.

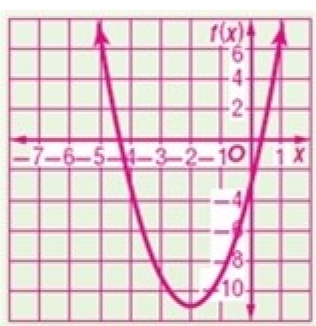
$f(x) = 2(0)^2 + 8(0) - 3$.
$f(x) = -3$
y-intercept $= -3$

The equation of axis of symmetry is $x = -\frac{b}{2a}$.

$$x = -\frac{8}{2 \cdot 2}$$
$$x = -2$$

The equation of the axis of symmetry is $x = -2$. Therefore, the x-coordinate of the vertex is -2.

Graph the function $f(x) = 2x^2 + 8x - 3$.



2. Determine whether $f(x) = 5 - x^2 + 2x$ has a maximum or a minimum value. Then find this maximum or minimum value and state the domain and range of the function.

SOLUTION:

For this function, $a = -1$, so the graph opens down and the function has a maximum value.

The maximum value of the function is the y-coordinate of the vertex.

The x-coordinate of the vertex is $-\frac{2}{2 \cdot -1}$ or 1.

Find the y-coordinate of the vertex by evaluating the function for $x = 1$.

$$\begin{aligned} f(x) &= 5 - 1^2 + 2(1) \\ &= 5 - 1 + 2 \\ &= 6 \end{aligned}$$

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $R = \{f(x) \mid f(x) \le 6\}$.

3. **MULTIPLE CHOICE** For which equation is the axis of symmetry $x = 5$?

A $f(x) = x^2 - 5x + 3$

B $f(x) = x^2 - 10x + 7$

C $f(x) = x^2 + 10x - 3$

D $f(x) = x^2 + 5x + 2$

SOLUTION:

Test option A.

Here, $a = 1$, $b = -5$, and $c = 3$.

The equation of axis of symmetry is $x = -\frac{b}{2a}$.

$$x = -\frac{-5}{2 \cdot 1}$$
$$= \frac{5}{2}$$

So, this option is incorrect.

Test option B.

Here, $a = 1$, $b = -10$, and $c = 7$.

The equation of axis of symmetry is $x = -\frac{b}{2a}$.

$$x = -\frac{-10}{2 \cdot 1}$$
$$x = 5$$

Therefore, option B is correct.

4. **PHYSICAL SCIENCE** From 4 feet above the ground, Maya throws a ball upward with a velocity of 18 feet per second. The height $h(t)$ of the ball t seconds after Maya throws the ball is given by $h(t) = -16t^2 + 18t + 4$. Find the maximum height reached by the ball and the time that this height is reached.

SOLUTION:

The maximum value of the function is the y-coordinate of the vertex.

The x-coordinate of the vertex is $-\frac{18}{2(-16)}$ or 0.5625.

Find the y-coordinate of the vertex by evaluating the function for $t = 0.5625$.

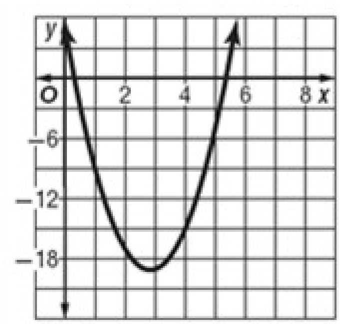
$$h(0.5625) = -16(0.5625)^2 + 18(0.5625) + 4$$
$$= 9.0625$$

The ball reached the maximum height of 9.0625 feet at 0.5625 seconds.

5. Solve $3x^2 - 17x + 5 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

SOLUTION:

Graph the function $f(x) = 3x^2 - 17x + 5$.



The x-intercepts of the graph indicate that one solution is between 0 and 1, and the other solution is between 5 and 6.

Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

6. Their sum is 15, and their product is 36.

SOLUTION:

Let x be a number and so the number is $(15 - x)$.

$$x(15-x)=36$$
$$15x-x^2=36$$
$$x^2-15x+36=0$$

Solve the quadratic equation $x^2-15x+36=0$.

$$x^2-15x+36=0$$
$$x^2-3x-12x+36=0$$
$$x(x-3)-12(x-3)=0$$
$$(x-3)(x-12)=0$$
$$\Rightarrow x=3 \text{ and } x=12$$

So, the two numbers are 3 and 12.

7. Their sum is 7, and their product is 15.

SOLUTION:

Let x, be the first number. Then $7 - x$ is the other number. $x(7 - x) = 15$; $-x^2 + 7x - 15 = 0$. Since the graph of the related function does not intersect the x-axis, this equation has no real solutions. Therefore, no such numbers exist.

8. **MULTIPLE CHOICE** Using the graph of the function $f(x) = x^2 + 6x - 7$, what are the solutions to the equation $x^2 + 6x - 7 = 0$?

F –1, 6

G 1, –6

H –1, 7

J 1, –7

SOLUTION:

The x-intercepts of the graph indicate that the solutions are –7 and 1.
So, the correct option is J.

9. **BASEBALL** A baseball is hit upward with a velocity of 40 feet per second. Ignoring the height of the baseball player, how long does it take for the ball to fall to the ground? Use the formula $h(t) = v_0t - 16t^2$ where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

SOLUTION:

Substitute $h(t) = 0$ and $v_0 = 40$ in $h(t) = v_0t - 16t^2$.

$$40t-16t^2=0$$
$$8t(5-2t)=0$$
$$t=0 \text{ or } 5-2t=0$$
$$t=2.5$$

So, the ball takes 2.5 seconds to fall to the ground.

Mid-Chapter Quiz: Lessons 4-1 through 4-4

Solve each equation by factoring.

10. $x^2 - x - 12 = 0$

SOLUTION:

$$\begin{aligned} x^2 - x - 12 &= 0 \\ x^2 - 4x + 3x - 12 &= 0 \\ x(x-4) + 3(x-4) &= 0 \\ (x+3)(x-4) &= 0 \\ &\Rightarrow x + 3 = 0 \text{ or } x - 4 = 0 \\ &\Rightarrow x = -3 \quad \text{or} \quad x = 4 \end{aligned}$$

Therefore, the roots are -3 and 4.

11. $3x^2 + 7x + 2 = 0$

SOLUTION:

$$\begin{aligned} 3x^2 + 7x + 2 &= 0 \\ 3x^2 + x + 6x + 2 &= 0 \\ x(3x+1) + 2(3x+1) &= 0 \\ (3x+1)(x+2) &= 0 \\ &\Rightarrow 3x + 1 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow x = -\frac{1}{3} \quad \text{or} \quad x = -2 \end{aligned}$$

Therefore, the roots are -2 and $-\frac{1}{3}$.

12. $x^2 - 2x - 15 = 0$

SOLUTION:

$$\begin{aligned} x^2 - 2x - 15 &= 0 \\ x^2 - 5x + 3x - 15 &= 0 \\ x(x-5) + 3(x-5) &= 0 \\ (x+3)(x-5) &= 0 \\ &\Rightarrow x + 3 = 0 \text{ or } x - 5 = 0 \\ &\Rightarrow x = -3 \quad \text{or} \quad x = 5 \end{aligned}$$

Therefore, the roots are -3 and 5.

13. $2x^2 + 5x - 3 = 0$

SOLUTION:

$$\begin{aligned} 2x^2 + 5x - 3 &= 0 \\ 2x^2 + 6x - x - 3 &= 0 \\ 2x(x+3) - 1(x+3) &= 0 \\ (2x-1)(x+3) &= 0 \\ &\Rightarrow 2x - 1 = 0 \text{ or } x + 3 = 0 \\ &\Rightarrow x = \frac{1}{2} \quad \text{or} \quad x = -3 \end{aligned}$$

Therefore, the roots are -3 and $\frac{1}{2}$.

14. Write a quadratic equation in standard form with roots –6 and $\frac{1}{4}$.

SOLUTION:

Write the pattern.

$$(x-p)(x-q) = 0$$

Replace p and q with -6 and $\frac{1}{4}$.

$$(x-(-6))\left(x - \frac{1}{4}\right) = 0$$

$$(x+6)\left(x - \frac{1}{4}\right) = 0$$

Use the FOIL method to multiply.

$$\begin{aligned} x(x) + x\left(-\frac{1}{4}\right) + 6(x) + 6\left(-\frac{1}{4}\right) &= 0 \\ x^2 - \frac{1}{4}x + 6x - \frac{3}{2} &= 0 \end{aligned}$$

Multiply each side by 4.

$$\begin{aligned} 4x^2 - x + 24x - 6 &= 0 \\ 4x^2 + 23x - 6 &= 0 \end{aligned}$$

15. **TRIANGLES** Find the dimensions of a triangle if the base is $\frac{2}{3}$ the measure of the height and the area is 12 square centimeters.

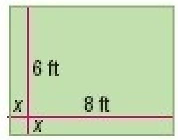
SOLUTION:
Let b and h be the base and height of the triangle.

$$b = \frac{2}{3}h$$
$$A = \frac{1}{2}bh$$
$$12 = \frac{1}{2}\left(\frac{2}{3}h\right)h$$
$$12 = \frac{1}{3}h^2$$
$$36 = h^2$$
$$h = \pm 6$$

The height cannot be negative, so take h = 6 cm.

$$b = \frac{2}{3}(6)$$
$$b = 4 \text{ cm}$$

16. **PATIO** Eli is putting a cement slab in his backyard. The original slab was going to have dimensions of 8 feet by 6 feet. He decided to make the slab larger by adding x feet to each side. The area of the new slab is 120 square feet.



a. Write a quadratic equation that represents the area of the new slab.

b. Find the new dimensions of the slab.

SOLUTION:
a.
$$(8+x)(6+x) = 120$$
$$(8)(6) + 6x + 8x + x^2 = 120$$
$$48 + 14x + x^2 = 120$$
$$48 + 14x + x^2 - 120 = 0$$
$$x^2 + 14x - 72 = 0$$

b.
$$x^2 + 14x - 72 = 0$$
$$x^2 + 18x - 4x - 72 = 0$$
$$x(x+18) - 4(x+18) = 0$$
$$(x+18)(x-4) = 0$$
$$\Rightarrow x + 18 = 0 \text{ or } x - 4 = 0$$
$$\Rightarrow x = -18 \quad \text{or} \quad x = 4$$

The value of x cannot be negative, so $x = 4$.

The dimensions of the slab are 12 feet by 10 feet.

Mid-Chapter Quiz: Lessons 4-1 through 4-4

Simplify.

17. $\sqrt{-81}$

SOLUTION:

$$\begin{aligned}\sqrt{-81} &= \sqrt{-1\cdot 9\cdot 9}\\ &= \sqrt{-1}\cdot\sqrt{9^2}\\ &= 9i\end{aligned}$$

18. $\sqrt{-25x^4y^5}$

SOLUTION:

$$\begin{aligned}\sqrt{-25x^4y^5} &= \sqrt{-1\cdot 5\cdot 5\cdot x^2\cdot x^2\cdot y^2\cdot y^2\cdot y}\\ &= \sqrt{-1}\cdot\sqrt{y}\cdot x^2y^2\\ &= 5x^2y^2i\sqrt{y}\end{aligned}$$

19. $(15-3i)-(4-12i)$

SOLUTION:

$$\begin{aligned}(15-3i)-(4-12i) &= 15-3i-4+12i\\ &= 11+9i\end{aligned}$$

20. i^{37}

SOLUTION:

$$\begin{aligned}i^{37} &= i^{36}\cdot i\\ &= \left(i^2\right)^{18}\cdot i\\ &= 1\cdot i\\ &= i\end{aligned}$$

21. $(5-3i)(5+3i)$

SOLUTION:

$$\begin{aligned}(5-3i)(5+3i) &= 5(5)+5(3i)-3i(5)-3i(3i)\\ &= 25+15i-15i-9i^2\\ &= 25+15i-15i-9(-1)\\ &= 25+9\\ &= 34\end{aligned}$$

22. $\dfrac{3-i}{2+5i}$

SOLUTION:

$$\begin{aligned}\frac{3-i}{2+5i} &= \frac{3-i}{2+5i}\cdot\frac{2-5i}{2-5i}\\ &= \frac{(3-i)(2-5i)}{(2+5i)(2-5i)}\\ &= \frac{6-15i-2i+5i^2}{4-25i^2}\\ &= \frac{6-17i-5}{4+25}\\ &= \frac{1-17i}{29}\\ &= \frac{1}{29}-\frac{17}{29}i\end{aligned}$$

23. The impedance in one part of a series circuit is $3 + 4j$ ohms and the impedance in another part of the circuit is $6 - 7j$ ohms. Add these complex numbers to find the total impedance in the circuit.

SOLUTION:

$$\begin{aligned}\text{Total impedance} &= 3+4j+6-7j\\ &= 9-3j\text{ ohms}\end{aligned}$$