Graph each equation.

20. $y = 3x^2 + 24x - 10$

SOLUTION:

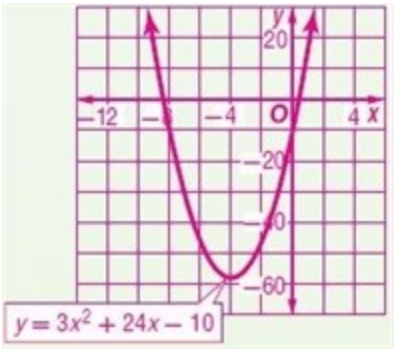
$$y = 3x^2 + 24x - 10$$
$$y + 10 = 3(x^2 + 8x)$$
$$y + 10 + 48 = 3(x^2 + 8x + 16)$$
$$y + 58 = 3(x + 4)^2$$

vertex: (–4, –58)
axis of symmetry: $x = -4$.

focus: $\left(-4, -57\frac{3}{4}\right)$

directrix: $y = -58\frac{1}{4}$

length of latus rectum: 1 unit
opens up



21. $3y - x^2 = 8x - 11$

SOLUTION:

$$3y - x^2 = 8x - 11$$
$$y + 11 = \frac{1}{3}(x^2 + 8x)$$
$$y + 11 + \frac{16}{3} = \frac{1}{3}(x^2 + 8x + 16)$$
$$y + \frac{49}{3} = \frac{1}{3}(x + 4)^2$$

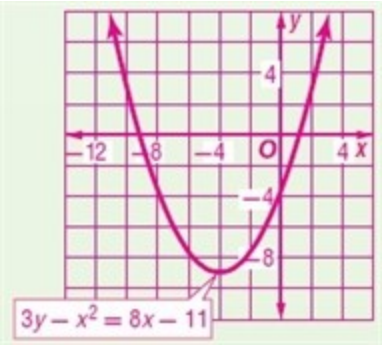
vertex: $\left(-4, -\frac{49}{3}\right)$

axis of symmetry: $x = -4$.

focus: $\left(-4, -15\frac{7}{12}\right)$

directrix: $x = -17\frac{1}{12}$

length of latus rectum: 3 unit
opens up

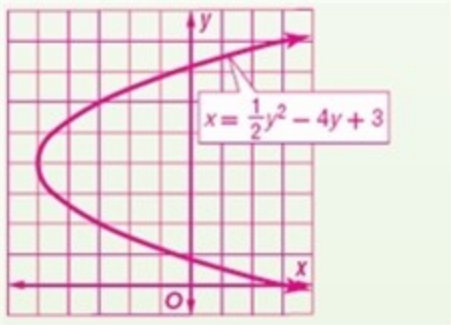


22. $x = \frac{1}{2}y^2 - 4y + 3$

SOLUTION:

$$x = \frac{1}{2}y^2 - 4y + 3$$
$$x - 3 = \frac{1}{2}(y^2 - 8y)$$
$$x - 3 + 8 = \frac{1}{2}(y^2 - 8y + 16)$$
$$x + 5 = \frac{1}{2}(y - 4)^2$$

vertex: $(-5, 4)$
axis of symmetry: $y = 4$.
focus: $\left(-4\frac{1}{2}, 4\right)$
directrix: $x = -5\frac{1}{2}$
length of latus rectum: 2 units
opens to the right

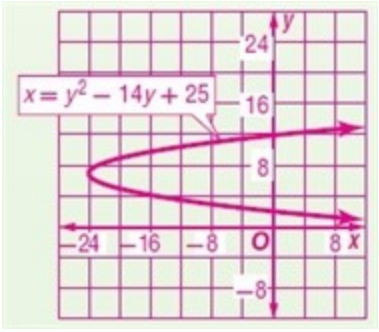


23. $x = y^2 - 14y + 25$

SOLUTION:

$$x = y^2 - 14y + 25$$
$$x - 25 = y^2 - 14y$$
$$x - 25 + 49 = y^2 - 14y + 49$$
$$x + 24 = (y - 7)^2$$
$$x = (y - 7)^2 - 24$$

vertex: (–24, 7)
axis of symmetry: $y = 7$.
focus: (–23.75, 7)
directrix: $x = -24.25$
length of latus rectum: 1 unit
opens to the right



Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

24. $y = \frac{1}{2}x^2$

SOLUTION:

$$y - 0 = \frac{1}{2}(x - 0)^2$$

vertex: (0, 0)
axis of symmetry: $x = 0$
opens up

25. $y = 4x^2 - 16x + 9$

SOLUTION:

$$\begin{aligned} y &= 4x^2 - 16x + 9 \\ y - 9 &= 4(x^2 - 4x) \\ y - 9 + 16 &= 4(x^2 - 4x + 4) \\ y + 7 &= 4(x - 2)^2 \\ y &= 4(x - 2)^2 - 7 \end{aligned}$$

vertex: (2, –7)
axis of symmetry: $x = 2$
opens up

26. $x - 6y = y^2 + 4$

SOLUTION:

$$\begin{aligned} x - 6y &= y^2 + 4 \\ x - 4 &= y^2 + 6y \\ x - 4 + 9 &= y^2 + 6y + 9 \\ x + 5 &= (y + 3)^2 \\ x &= (y + 3)^2 - 5 \end{aligned}$$

vertex: (–5, –3)
axis of symmetry: $y = -3$
opens to the right

27. $x = y^2 + 14y + 20$

SOLUTION:

$$\begin{aligned} x &= y^2 + 14y + 20 \\ x - 20 &= y^2 + 14y \\ x - 20 + 49 &= y^2 + 14y + 49 \\ x + 29 &= (y + 7)^2 \\ x &= (y + 7)^2 - 29 \end{aligned}$$

vertex: (–29, –7)
axis of symmetry: $y = -7$
opens to the right

28. **SPORTS** When a football is kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 50 feet, and lands 200 feet away. Assuming the football was kicked at the origin, write an equation of the parabola that models the flight of the football.

SOLUTION:

The coordinates of the ball when the ball touches the ground are (0, 0) and (200, 0).
The vertex of the path is (100, 50).
The standard form of equation of a parabola is $y = a(x - h)^2 + k$.
Substitute 0, 0 , 100 and 50 for y, x, h and k in the standard form then solve for a.

$$\begin{aligned} 0 &= a(0 - 100)^2 + 50 \\ 10000a &= -50 \\ a &= -\frac{1}{200} \end{aligned}$$

The equation of the path is $y = -\frac{1}{200}(x - 100)^2 + 50$.

Write an equation for the circle that satisfies each set of conditions.

29. center (–1, 6), radius 3 units

SOLUTION:

Substitute –1, 6 and 3 for h, k and d in the standard form of circle equation.

$$\begin{aligned} (x - (-1))^2 + (y - 6)^2 &= (3)^2 \\ (x + 1)^2 + (y - 6)^2 &= 9 \end{aligned}$$

30. endpoints of a diameter (2, 5) and (0, 0)

SOLUTION:

The center is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$(h, k) = \left(\frac{2-0}{2}, \frac{5-0}{2}\right)$$
$$= (1, 2.5)$$

The radius of the circle is
$$\frac{\sqrt{(2-0)^2 + (5-0)^2}}{2} = \frac{\sqrt{29}}{2}$$

Substitute 1, 2.5 and $\frac{\sqrt{29}}{2}$ for h, k and d in the standard form of circle equation.

$$(x-1)^2 + (y-2.5)^2 = \left(\frac{\sqrt{29}}{2}\right)^2$$
$$(x-1)^2 + (y-2.5)^2 = \frac{29}{4}$$

31. endpoints of a diameter (4, –2) and (–2, –6)

SOLUTION:

$$(h, k) = \left(\frac{4-2}{2}, \frac{-2-6}{2}\right)$$
$$= (1, -4)$$

$$d = \sqrt{(1+2)^2 + (-4+6)^2}$$
$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

The radius of the circle is $\sqrt{13}$ units.
Substitute 1, –4 and 5 for h, k and d in the standard form of circle equation.

$$(x-1)^2 + (y-(-4))^2 = \left(\sqrt{13}\right)^2$$
$$(x-1)^2 + (y+4)^2 = 13$$

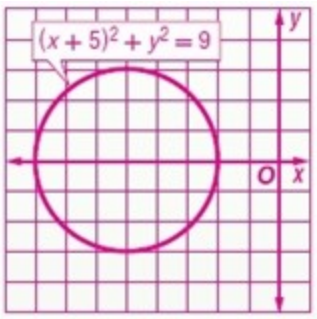
Find the center and radius of each circle. Then graph the circle.

32. $(x+5)^2 + y^2 = 9$

SOLUTION:

$$(x+5)^2 + y^2 = 9$$
$$(x-(-5))^2 + (y-0)^2 = 3^2$$

The center of the circle is (–5, 0).
The radius of the circle is 3.

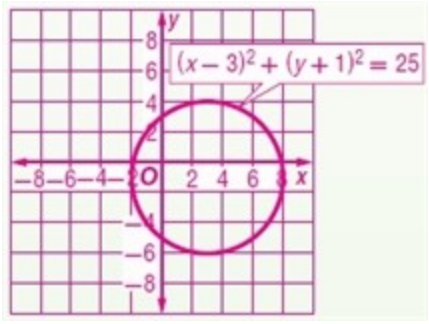


33. $(x-3)^2+(y+1)^2=25$

SOLUTION:

$$(x-3)^2+(y+1)^2=25$$
$$(x-3)^2+(y-(-1))^2=5^2$$

The center of the circle is (3, –1).
The radius of the circle is 5.

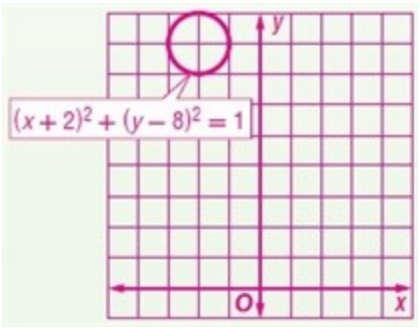


34. $(x+2)^2+(y-8)^2=1$

SOLUTION:

$$(x+2)^2+(y-8)^2=1$$
$$(x-(-2))^2+(y-8)^2=1^2$$

The center of the circle is (–2, 8).
The radius of the circle is 1.

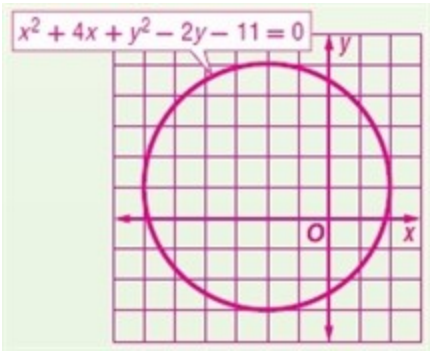


35. $x^2+4x+y^2-2y-11=0$

SOLUTION:

$$x^2+4x+y^2-2y-11=0$$
$$x^2+4x+y^2-2y=11$$
$$(x^2+4x+4)+(y^2-2y+1)=11+4+1$$
$$(x+2)^2+(y-1)^2=16$$
$$(x-(-2))^2+(y-1)^2=4^2$$

The center of the circle is (–2, 1).
The radius of the circle is 4.



36. **SOUND** A loudspeaker in a school is located at the point (65, 40). The speaker can be heard in a circle with a radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

SOLUTION:

The center of the circle is (65, 40) and the radius is 100.
Substitute 65, 40 and 100 for h, k and r in the standard form of the circle equation.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-65)^2+(y-40)^2=100^2$$