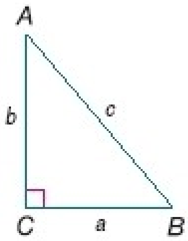
Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



7. $c = 12$, $b = 5$

SOLUTION:

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin B}{5} = \frac{\sin 90^\circ}{12}$$
$$\sin B = \frac{5 \sin 90^\circ}{12}$$
$$B \approx 25^\circ$$
$$m\angle A \approx 180^\circ - (90^\circ + 25^\circ) \text{ or } 65^\circ$$

Use the Law of Sines to find the side length a.

$$\frac{\sin A}{a} \approx \frac{\sin C}{c}$$
$$\frac{\sin 65^\circ}{a} \approx \frac{\sin 90^\circ}{12}$$
$$a \approx 10.9$$

8. $a = 10$, $B = 55°$

SOLUTION:

$$m\angle A = 180^\circ - (90^\circ + 55^\circ) \text{ or } 35^\circ$$

Use the Law of Sines to find the side lengths b and c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 35^\circ}{10} = \frac{\sin 90^\circ}{c}$$
$$c = \frac{10 \sin 90^\circ}{\sin 35^\circ}$$
$$c \approx 17.4$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 35^\circ}{10} = \frac{\sin 55^\circ}{b}$$
$$b = \frac{10 \sin 55^\circ}{\sin 35^\circ}$$
$$b \approx 14.3$$

9. $B = 75°$, $b = 15$

SOLUTION:

$m\angle A = 180° - (90° + 75°)$ or $15°$

Use the Law of Sines to find the side lengths a and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 15°}{a} = \frac{\sin 75°}{15}$$
$$a = \frac{15 \sin 15°}{\sin 75°}$$
$$a \approx 4.0$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 90°}{c} = \frac{\sin 75°}{15}$$
$$c = \frac{15 \sin 90°}{\sin 75°}$$
$$c \approx 15.5$$

10. $B = 45°$, $c = 16$

SOLUTION:

$m\angle A = 180° - (90° + 45°)$ or $45°$

Use the Law of Sines to find the side lengths a and b.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 90°}{16} = \frac{\sin 45°}{b}$$
$$b = \frac{16 \sin 45°}{\sin 90°}$$
$$b \approx 11.3$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 90°}{16} = \frac{\sin 45°}{a}$$
$$a = \frac{16 \sin 45°}{\sin 90°}$$
$$a \approx 11.3$$

11. $A = 35°$, $c = 22$

SOLUTION:

$m\angle B = 180^\circ - (90^\circ + 35^\circ)$ or 55°

Use the Law of Sines to find the side lengths *a* and *b*.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 90^\circ}{22} = \frac{\sin 55^\circ}{b}$$
$$b = \frac{22 \sin 55^\circ}{\sin 90^\circ}$$
$$b \approx 18.0$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 90^\circ}{22} = \frac{\sin 35^\circ}{a}$$
$$a = \frac{22 \sin 35^\circ}{\sin 90^\circ}$$
$$a \approx 12.6$$

12. $\sin A = \frac{2}{3}$, $a = 6$

SOLUTION:

$$\sin A = \frac{2}{3}$$
$$A \approx 42^\circ$$
$$m\angle B \approx 180^\circ - (90^\circ + 42^\circ) \text{ or } 48^\circ$$

Use the Law of Sines to find the side lengths *b* and *c*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 42^\circ}{6} \approx \frac{\sin 48^\circ}{b}$$
$$b \approx \frac{6 \sin 48^\circ}{\sin 42^\circ}$$
$$b \approx 6.7$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 42^\circ}{6} \approx \frac{\sin 90^\circ}{c}$$
$$c \approx \frac{6 \sin 90^\circ}{\sin 42^\circ}$$
$$c \approx 9.0$$

13. **TRUCK** The back of a moving truck is 3 feet off of the ground. What length does a ramp off the back of the truck need to be in order for the angle of elevation of the ramp to be 20°?

SOLUTION:
Draw the diagram which represents the situation.

Let x be the length of the ramp off.

Use the Law of Sines to find the length of x.

$$\frac{\sin 90^\circ}{x} \approx \frac{\sin 20^\circ}{3}$$
$$x \approx \frac{3\sin 90^\circ}{\sin 20^\circ}$$
$$x \approx 8.8\text{ ft}$$

Rewrite each degree measure in radians and each radian measure in degrees.

14. 215°

SOLUTION:

$$215^\circ = 215^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$$
$$= \frac{215\pi}{180} \text{ or } \frac{43\pi}{36} \text{radians}$$

15. $\frac{5\pi}{2}$

SOLUTION:

$$\frac{5\pi}{2} = \frac{5\pi}{2}\text{radians} \cdot \frac{180^\circ}{\pi \text{radians}}$$
$$= \frac{5 \cdot 180^\circ}{2}$$
$$= 450^\circ$$

16. −3π

SOLUTION:

$$-3\pi = -3\pi \text{ radians} \cdot \frac{180^\circ}{\pi \text{radians}}$$
$$= -3 \cdot 180^\circ$$
$$= -540^\circ$$

17. −315°

SOLUTION:

$$-315^\circ = -315^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$$
$$= -\frac{315\pi}{180} \text{ or } -\frac{7\pi}{4}\text{radians}$$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

18. 265°

SOLUTION:
Positive angle: 265° + 360° = 625°

Negative angle: 265° – 360° = –95°

19. −65°

SOLUTION:
Positive angle: –65° + 360° = 295°

Negative angle: –65° – 360° = –425°

20. $\frac{7\pi}{2}$

SOLUTION:
Positive angle: $\frac{7\pi}{2} + 2\pi = \frac{11\pi}{2}$

Negative angle: $\frac{7\pi}{2} - 4\pi = -\frac{\pi}{2}$

21. **BICYCLE** A bicycle tire makes 8 revolutions in one minute. The tire has a radius of 15 inches. Find the angle θ in radians through which the tire rotates in one second.

SOLUTION:
One revolution makes 360°, so 8 revolutions make 2880°(360° × 8).
For the rotation of 2880° it takes 60 seconds.

Therefore, for one sec it makes $\frac{2880^\circ}{60}$ or 48° angle of rotation.

Rewrite 48° in radians.

$$48^\circ = 48^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$$
$$= \frac{48\pi}{180} \text{ or } \frac{4\pi}{15} \text{ radians}$$

Find the exact value of each trigonometric function.

22. cos 135°

SOLUTION:
The terminal side of 135º lies in Quadrant II.

$$\theta' = 180^\circ - \theta$$
$$= 180^\circ - 135^\circ$$
$$= 45^\circ$$

The cosine function is negative in Quadrant II.

$$\cos 150^\circ = -\cos 45^\circ$$
$$= -\frac{\sqrt{2}}{2}$$

23. tan 150°

SOLUTION:
The terminal side of 150º lies in Quadrant II.

$$\theta' = 180^\circ - \theta$$
$$= 180^\circ - 150^\circ$$
$$= 30^\circ$$

The tangent function is negative in Quadrant II.

$$\tan 150^\circ = -\tan 30^\circ$$
$$= -\frac{\sqrt{3}}{3}$$

24. sin 2π

SOLUTION:

Since the angle 2π is a quadrant angle, the coordinates of the point on its terminal side is (*x*, 0).

Find the value of *r*.

$$r = \sqrt{a^2 + b^2}$$
$$= \sqrt{(x)^2 + 0^2}$$
$$r = x$$

$$\sin 2\pi = \frac{y}{r}$$
$$= \frac{0}{x}$$
$$= 0$$

25. $\cos \frac{3\pi}{2}$

SOLUTION:

Since the angle $\frac{3\pi}{2}$ is a quadrant angle, the coordinates of the point on its terminal side is (0, –*y*).

Find the value of *r*.

$$r = \sqrt{a^2 + b^2}$$
$$= \sqrt{0^2 + (-y)^2}$$
$$r = y$$

$$\cos \frac{3\pi}{2} = \frac{x}{r}$$
$$= \frac{0}{y}$$
$$= 0$$

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

26. *P*(−4, 3)

SOLUTION:

Find the value of *r*

.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-4)^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5$$

Use *x* = –4, *y* = 3, and *r* = 5 to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{3}{5}$$
$$\cos\theta = \frac{x}{r} = -\frac{4}{5}$$
$$\tan\theta = \frac{y}{x} = -\frac{3}{4}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{3}$$
$$\sec\theta = \frac{r}{x} = -\frac{5}{4}$$
$$\cot\theta = \frac{x}{y} = -\frac{4}{3}$$

27. $P(5, 12)$

SOLUTION:
Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Use $x = 5, y = 12$, and $r = 13$ to write the six trigonometric ratios.

$$\begin{aligned} \sin\theta &= \frac{y}{r} = \frac{12}{13} \\ \cos\theta &= \frac{x}{r} = \frac{5}{13} \\ \tan\theta &= \frac{y}{x} = \frac{12}{5} \\ \csc\theta &= \frac{r}{y} = \frac{13}{12} \\ \sec\theta &= \frac{r}{x} = \frac{13}{5} \\ \cot\theta &= \frac{x}{y} = \frac{5}{12} \end{aligned}$$

28. $P(16, -12)$

SOLUTION:
Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{16^2 + (-12)^2} \\ &= \sqrt{400} \\ &= 20 \end{aligned}$$

Use $x = 16, y = -12$, and $r = 20$ to write the six trigonometric ratios.

$$\begin{aligned} \sin\theta &= \frac{y}{r} = -\frac{3}{5} \\ \cos\theta &= \frac{x}{r} = \frac{4}{5} \\ \tan\theta &= \frac{y}{x} = -\frac{3}{4} \\ \csc\theta &= \frac{r}{y} = -\frac{5}{3} \\ \sec\theta &= \frac{r}{x} = \frac{5}{4} \\ \cot\theta &= \frac{x}{y} = -\frac{4}{3} \end{aligned}$$

29. **BALL** A ball is thrown off the edge of a building at an angle of 70° and with an initial velocity of 5 meters per second. The equation that represents the horizontal distance of the ball x is $x = v_0(\cos\theta)t$, where v_0 is the initial velocity, θ is the angle at which it is thrown, and t is the time in seconds. About how far will the ball travel in 10 seconds?

SOLUTION:
Substitute 5 for v_0, 70° for θ, and 10 for t in the given equation and solve for x.

$$\begin{aligned} x &= v_0(\cos\theta)t \\ x &= 5(\cos 70^\circ)10 \\ &= 50(\cos 70^\circ) \\ &\approx 17.1 \text{ meters} \end{aligned}$$

Find the exact value of each function.

40. cos (−210°)

SOLUTION:

$$\begin{aligned}\cos(-210°) &= \cos 210° \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

41. (cos 45°)(cos 210°)

SOLUTION:

$$\begin{aligned}(\cos 45°)(\cos 210°) &= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) \\ &= -\frac{\sqrt{6}}{4}\end{aligned}$$

42. $\sin -\frac{7\pi}{4}$

SOLUTION:

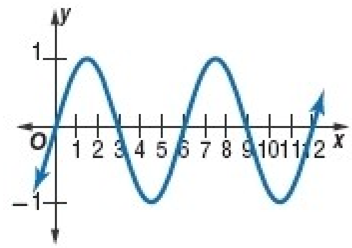
$$\begin{aligned}\sin\left(-\frac{7\pi}{4}\right) &= -\sin\frac{7\pi}{4} \\ &= -\left(-\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

43. $\left(\cos\frac{\pi}{2}\right)\left(\sin\frac{\pi}{2}\right)$

SOLUTION:

$$\begin{aligned}\left(\cos\frac{\pi}{2}\right)\left(\sin\frac{\pi}{2}\right) &= (0)(1) \\ &= 0\end{aligned}$$

44. Determine the period of the function.



SOLUTION:

The pattern repeats at 6, 12 and so on. So, the period of the function is 6.

45. A wheel with a diameter of 18 inches completes 4 revolutions in 1 minute. What is the period of the function that describes the height of one spot on the outside edge of the wheel as a function of time?

SOLUTION:

Period:
Since the wheel completes 4 revolutions in 1 minute, the time taken to make one revolution is $\frac{60}{4}$ or 15 seconds. So, the period is 15 seconds.

Find the amplitude, if it exists, and period of each function. Then graph the function.

46. $y = 4 \sin 2\theta$

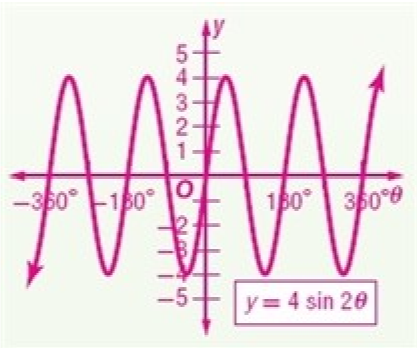
SOLUTION:

Amplitude: $|a| = |4|$ or 4

Period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{|2|}$ or 180°

x-intercepts: (0,0)

$$\left(\frac{1}{2} \cdot \frac{360^\circ}{b}, 0\right) = \left(\frac{1}{2} \cdot \frac{360^\circ}{2}, 0\right) \text{ or } (90^\circ, 0)$$
$$\left(\frac{360^\circ}{b}, 0\right) = \left(\frac{360^\circ}{2}, 0\right) \text{ or } (180^\circ, 0)$$



47. $y = \cos\frac{1}{2}\theta$

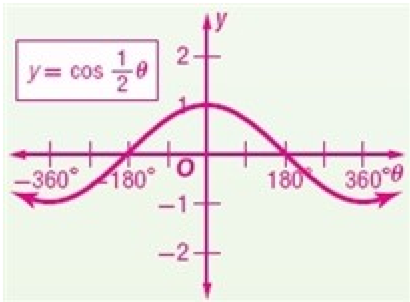
SOLUTION:

Amplitude: $|a| = |1|$ or 1

Period: $\frac{360^\circ}{|b|} = \frac{360^\circ}{\left|\frac{1}{2}\right|}$ or 720°

x-intercepts:

$$\left(\frac{1}{4} \cdot \frac{360^\circ}{b}, 0\right) = \left(\frac{1}{4} \cdot \frac{360^\circ}{\frac{1}{2}}, 0\right) \text{ or } (180^\circ, 0)$$
$$\left(\frac{3}{4} \cdot \frac{360^\circ}{b}, 0\right) = \left(\frac{3}{4} \cdot \frac{360^\circ}{\frac{1}{2}}, 0\right) \text{ or } (540^\circ, 0)$$

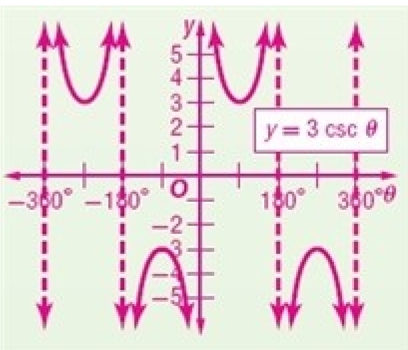


48. $y = 3 \csc \theta$

SOLUTION:
Amplitude: not defined

Period: $\frac{360°}{|b|} = \frac{360°}{|1|}$ or $360°$

The vertical asymptotes occur at the points where $3\sin\theta = 0$. So, the asymptotes are at $\theta = 180°$ and $\theta = 360°$. Sketch $y = 3\sin\theta$ and use it to graph $y = 3\csc\theta$.

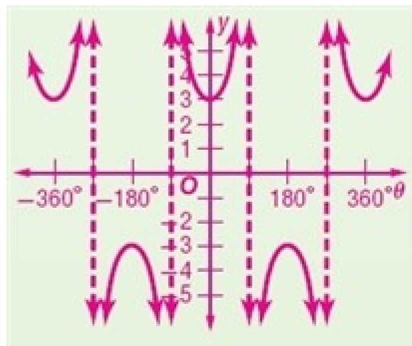


49. $y = 3 \sec \theta$

SOLUTION:
Amplitude: not defined

Period: $\frac{360°}{|b|} = \frac{360°}{|1|}$ or $360°$

The vertical asymptotes occur at the points where $3\cos\theta = 0$. So, the asymptotes are at $\theta = 90°$ and $\theta = 270°$. Sketch $y = 3\cos\theta$ and use it to graph $y = 3\sec\theta$.

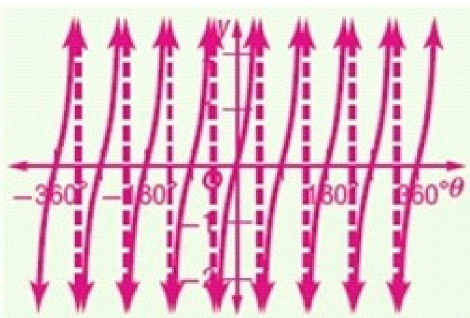


50. $y = \tan 2\theta$

SOLUTION:
Amplitude: not defined

Period: $\dfrac{180^\circ}{|b|} = \dfrac{180^\circ}{|2|}$ or 90°

The vertical asymptotes occur at the points where $\tan 2\theta = 0$. So, the asymptotes are at $\theta = 45^\circ$ and $\theta = 135^\circ$. Sketch $y = 3\cos\theta$ and use it to graph $y = \tan 2\theta$.

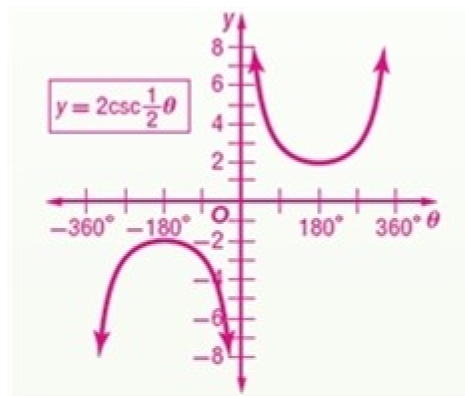


51. $y = 2\csc\dfrac{1}{2}\theta$

SOLUTION:
Amplitude: not defined

Period: $\dfrac{360^\circ}{|b|} = \dfrac{360^\circ}{\left|\frac{1}{2}\right|}$ or 720°

The vertical asymptotes occur at the points where $2\sin\dfrac{1}{2}\theta = 0$. So, the asymptotes are at θ = 360º and θ = 720º. Sketch $y = 2\sin\dfrac{1}{2}\theta$ and use it to graph $y = 2\csc\dfrac{1}{2}\theta$.



52. When Lauren jumps on a trampoline it vibrates with a frequency of 10 hertz. Let the amplitude equal 5 feet. Write a sine equation to represent the vibration of the trampoline y as a function of time *t*.

SOLUTION:

The amplitude of the function is 5.

Since the period is the reciprocal of the frequency, the period of the function is $\frac{1}{10}$.

Period = $\frac{2\pi}{|b|}$

$$\frac{1}{10} = \frac{2\pi}{|b|}$$
$$|b| = 20\pi$$
$$b = \pm 20\pi$$

Substitute 5 for a, 20π for b and t for θ in the general equation for the sine function.

$$y = a \sin b\theta$$
$$y = 5 \sin 20\pi t$$