

Study Guide and Review - Chapter 10

Find the indicated term of each arithmetic sequence.

11. $a_1 = 9, d = 3, n = 14$

SOLUTION:

Substitute 9 for a_1 , 3 for d , and 14 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_{14} &= 9 + (14-1)3 \\&= 9 + 39 \\&= 48\end{aligned}$$

12. $a_1 = -3, d = 6, n = 22$

SOLUTION:

Substitute -3 for a_1 , 6 for d , and 22 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_{22} &= -3 + (22-1)6 \\&= -3 + 126 \\&= 123\end{aligned}$$

13. $a_1 = 10, d = -4, n = 9$

SOLUTION:

Substitute 10 for a_1 , -4 for d , and 9 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_9 &= 10 + (9-1)(-4) \\&= 10 - 32 \\&= -22\end{aligned}$$

14. $a_1 = -1, d = -5, n = 18$

SOLUTION:

Substitute -1 for a_1 , -5 for d , and 18 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_{18} &= -1 + (18-1)(-5) \\&= -1 - 85 \\&= -86\end{aligned}$$

Find the arithmetic means in each sequence.

15. $-12, \underline{\quad}, \underline{\quad}, \underline{\quad}, 8$

SOLUTION:

Substitute -12 for a_1 , 5 for n , and 8 for a_5 in the formula for the n th term and find d .

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_5 &= -12 + (5-1)d \\8 &= -12 + 4d \\4d &= 20 \\d &= 5\end{aligned}$$

The arithmetic means are $(-12 + 5)$ or -7 , $(-7 + 5)$ or -2 and $(-2 + 5)$ or 3.

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16. 15, __, __, 29

SOLUTION:

Substitute 15 for a_1 , 4 for n , and 29 for a_4 in the formula for the n th term and find d .

$$a_n = a_1 + (n-1)d$$

$$a_4 = 15 + (4-1)d$$

$$29 = 15 + 3d$$

$$3d = 14$$

$$d = \frac{14}{3}$$

The arithmetic means are

$$\left(15 + \frac{14}{3}\right) \text{ or } \frac{59}{3}, \left(\frac{59}{3} + \frac{14}{3}\right) \text{ or } \frac{73}{3}.$$

17. 12, __, __, __, __, -8

SOLUTION:

Substitute 12 for a_1 , 6 for n , and -8 for a_6 in the formula for the n th term and find d .

$$a_n = a_1 + (n-1)d$$

$$a_6 = 12 + (6-1)d$$

$$-8 = 12 + 5d$$

$$5d = -20$$

$$d = -4$$

The arithmetic means are $(12 - 4)$ or 8, $(8 - 4)$ or 4, $(4 - 4)$ or 0 and $(0 - 4)$ or -4 .

18. 72, __, __, __, 24

SOLUTION:

Substitute 72 for a_1 , 5 for n , and 24 for a_5 in the formula for the n th term and find d .

$$a_n = a_1 + (n-1)d$$

$$a_5 = 72 + (5-1)d$$

$$24 = 72 + 4d$$

$$4d = -48$$

$$d = -12$$

The arithmetic means are $(72 - 12)$ or 60, $(60 - 12)$ or 48 and $(48 - 12)$ or 36.

19. **BANKING** Carson saves \$40 every 2 months. If he saves at this rate for two years, how much will he have at the end of two years?

SOLUTION:

Substitute 40 for a_1 , 12 for n and 40 for d in the formula for the n th term and find a_{12} .

$$a_n = a_1 + (n-1)d$$

$$a_{12} = 40 + (12-1)40$$

$$= 40 + 440$$

$$= \$480$$

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Find S_n for each arithmetic series.

20. $a_1 = 16$, $a_n = 48$, $n = 6$

SOLUTION:

Substitute 16 for a_1 , 48 for a_n , 6 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) \\ &= \frac{6}{2}(16 + 48) \\ &= 3(64) \\ &= 192 \end{aligned}$$

21. $a_1 = 8$, $a_n = 96$, $n = 20$

SOLUTION:

Substitute 8 for a_1 , 96 for a_n , 20 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) \\ &= \frac{20}{2}(8 + 96) \\ &= 10(104) \\ &= 1040 \end{aligned}$$

22. $9 + 14 + 19 + \dots + 74$

SOLUTION:

Substitute 9 for a_1 , 5 for d , and 74 for a_n in the formula for the n th term and find n .

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ 74 &= 9 + (n-1)5 \\ 74 &= 9 + 5n - 5 \\ 5n &= 70 \\ n &= 14 \end{aligned}$$

Substitute 9 for a_1 , 74 for a_n , 14 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) \\ &= \frac{14}{2}(9 + 74) \\ &= 7(83) \\ &= 581 \end{aligned}$$

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23. $16 + 7 + -2 + \dots + -65$

SOLUTION:

Substitute 16 for a_1 , -9 for d , and -65 for a_n in the formula for the n th term and find n .

$$\begin{aligned}a_n &= a_1 + (n-1)d \\-65 &= 16 + (n-1)(-9) \\-65 &= 16 - 9n + 9 \\9n &= 90 \\n &= 10\end{aligned}$$

Substitute 16 for a_1 , -65 for a_n , 10 for n in the Sum formula.

$$\begin{aligned}S_n &= \frac{n}{2}(a_1 + a_n) \\&= \frac{10}{2}(16 - 65) \\&= 5(-49) \\&= -245\end{aligned}$$

24. **DRAMA** Laura has a drama performance in 12 days. She plans to practice her lines each night. On the first night she rehearses her lines 2 times. The next night she rehearses her lines 4 times. The third night she rehearses her lines 6 times. On the eleventh night, how many times has she rehearsed her lines?

SOLUTION:

The arithmetic sequence that represents the situation is 2, 4, 6, ...

Substitute 2 for a_1 , 2 for d , and 11 for n in the formula for the n th term and find a_{11} .

$$\begin{aligned}a_n &= a_1 + (n-1)d \\a_{11} &= 2 + (11-1)2 \\&= 2 + 20 \\&= 22\end{aligned}$$

Substitute 2 for a_1 , 22 for a_n , 11 for n in the Sum formula.

$$\begin{aligned}S_n &= \frac{n}{2}(a_1 + a_n) \\&= \frac{11}{2}(2 + 22) \\&= \frac{11}{2}(24) \\&= 132\end{aligned}$$

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Find the sum of each arithmetic series.

$$25. \sum_{k=5}^{21} (3k - 2)$$

SOLUTION:

Use the formula $S_n = \frac{n}{2}(a_1 + a_n)$.

There are 17 terms, $a_1 = 3(5) - 2$ or 13, and $a_{21} = 3(21) - 2$ or 61.

$$\begin{aligned} S_{17} &= \frac{17}{2}(13 + 61) \\ &= 629 \end{aligned}$$

$$26. \sum_{k=0}^{10} (6k - 1)$$

SOLUTION:

Use the formula $S_n = \frac{n}{2}(a_1 + a_n)$.

There are 11 terms, $a_1 = 6(0) - 1$ or -1 , and $a_{11} = 6(10) - 1$ or 59.

$$\begin{aligned} S_{11} &= \frac{11}{2}(-1 + 59) \\ &= 319 \end{aligned}$$

$$27. \sum_{k=4}^{12} (-2k + 5)$$

SOLUTION:

Use the formula $S_n = \frac{n}{2}(a_1 + a_n)$.

There are 9 terms, $a_1 = -2(4) + 5$ or -3 , and $a_9 = -2(12) + 5$ or -19 .

$$\begin{aligned} S_9 &= \frac{9}{2}(-3 - 19) \\ &= -99 \end{aligned}$$

Find the indicated term for each geometric sequence.

$$28. a_1 = 5, r = 2, n = 7$$

SOLUTION:

Substitute 5 for a_1 , 2 for r , 7 for n in the formula for the n th term and simplify.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} \\ a_7 &= 5 \cdot (2^{7-1}) \\ &= 5(64) \\ &= 320 \end{aligned}$$

The seventh term is 320.

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29. $a_1 = 11, r = 3, n = 3$

SOLUTION:

Substitute 11 for a_1 , 3 for r , 3 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 \cdot r^{n-1} \\a_3 &= 11 \cdot (3^{3-1}) \\&= 11(9) \\&= 99\end{aligned}$$

The third term is 99.

30. $a_1 = 128, r = -\frac{1}{2}, n = 5$

SOLUTION:

Substitute 128 for a_1 , $-\frac{1}{2}$ for r , 5 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 \cdot r^{n-1} \\a_5 &= 128 \cdot \left(-\frac{1}{2}\right)^{5-1} \\&= 8\end{aligned}$$

The fifth term is 8.

31. a_8 for $\frac{1}{8}, \frac{3}{8}, \frac{9}{8}, \dots$

SOLUTION:

Find the common ratio r .

$$\begin{aligned}r &= \frac{3}{8} \div \frac{1}{8} \\&= \frac{3}{8} \times \frac{8}{1} \\&= 3\end{aligned}$$

Substitute $\frac{1}{8}$ for a_1 , 3 for r , 8 for n in the formula for the n th term and simplify.

$$\begin{aligned}a_n &= a_1 \cdot r^{n-1} \\a_8 &= \frac{1}{8} \cdot 3^{8-1} \\&= \frac{2187}{8}\end{aligned}$$

The eighth term is $\frac{2187}{8}$.

Find the geometric means in each sequence.

32. 6, __, __, 162

SOLUTION:

Substitute 6 for a_1 , 162 for a_4 , 4 for n in the formula for the n th term and find r .

$$\begin{aligned}a_n &= a_1 \cdot r^{n-1} \\a_4 &= 6 \cdot r^{4-1} \\162 &= 6r^3 \\27 &= r^3 \\3 &= r\end{aligned}$$

The geometric means are 6(3) or 18 and 18(3) or 54.

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33. 8, __, __, __, 648

SOLUTION:

Substitute 8 for a_1 , 648 for a_5 , 5 for n in the formula for the n th term and find r .

$$a_n = a_1 \cdot r^{n-1}$$

$$a_5 = 8 \cdot r^{5-1}$$

$$648 = 8r^4$$

$$81 = r^4$$

$$\pm 3 = r$$

The geometric means are $8(\pm 3)$ or ± 24 , $\pm 24(\pm 3)$ or 72 and $72(\pm 3)$ or ± 216 .

34. -4 , __, __, 108

SOLUTION:

Substitute -4 for a_1 , 108 for a_4 , 4 for n in the formula for the n th term and find r .

$$a_n = a_1 \cdot r^{n-1}$$

$$a_4 = -4 \cdot r^{4-1}$$

$$108 = -4r^3$$

$$-27 = r^3$$

$$-3 = r$$

The geometric means are $-4(-3)$ or 12 and $12(-3)$ or -36 .

35. **SAVINGS** Nolan has a savings account with a current balance of \$1500. What would be Nolan's account balance after 4 years if he receives 5% interest annually?

SOLUTION:

Substitute 1500 for a_0 , 1.05 for r and 4 for n in the formula $a_n = a_0 r^n$ and simplify.

$$a_n = a_0 r^n$$

$$a_4 = 1500(1.05^4)$$

$$\approx \$1823.26$$

Find S_n for each geometric series.

36. $a_1 = 15$, $r = 2$, $n = 4$

SOLUTION:

Substitute 15 for a_1 , 2 for r , 4 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{15 - 15(2^4)}{1 - 2} \\ &= \frac{-225}{-1} \\ &= 225 \end{aligned}$$

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37. $a_1 = 9, r = 4, n = 6$

SOLUTION:

Substitute 9 for a_1 , 4 for r , 6 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{9 - 9(4^6)}{1 - 4} \\ &= \frac{-36855}{-3} \\ &= 12,285 \end{aligned}$$

38. $5 - 10 + 20 - \dots$ to 7 terms

SOLUTION:

Find the common ratio r .

$$\begin{aligned} r &= \frac{-10}{5} \\ &= -2 \end{aligned}$$

Substitute 5 for a_1 , -2 for r , 7 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{5 - 5(-2^7)}{1 - (-2)} \\ &= \frac{645}{3} \\ &= 215 \end{aligned}$$

39. $243 + 81 + 27 + \dots$ to 5 terms

SOLUTION:

Find the common ratio r .

$$\begin{aligned} r &= \frac{81}{243} \\ &= \frac{1}{3} \end{aligned}$$

Substitute 243 for a_1 , $\frac{1}{3}$ for r , 5 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{243 - 243 \cdot \left(\frac{1}{3}\right)^5}{1 - \frac{1}{3}} \\ &= 242 \div \frac{2}{3} \\ &= 363 \end{aligned}$$

Evaluate the sum of each geometric series.

40. $\sum_{k=1}^7 3 \cdot (-2)^{k-1}$

SOLUTION:

Substitute 3 for a_1 , -2 for r , 7 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{3 - 3(-2)^7}{1 - (-2)} \\ &= \frac{387}{3} \\ &= 129 \end{aligned}$$

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$$41. \sum_{k=1}^8 -1 \left(\frac{2}{3}\right)^{k-1}$$

SOLUTION:

Substitute -1 for a_1 , $\frac{2}{3}$ for r , 8 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{-1 + 1 \cdot \left(\frac{2}{3}\right)^8}{1 - \frac{2}{3}} \\ &= -1 + \frac{256}{6561} \div \frac{1}{3} \\ &= -\frac{6305}{6561} \times 3 \\ &= -\frac{6305}{2187} \end{aligned}$$

42. **ADVERTISING** Natalie is handing out fliers to advertise the next student council meeting. She hands out fliers to 4 people. Then, each of those 4 people hand out 4 fliers to 4 other people. Those 4 then hand out 4 fliers to 4 new people. If Natalie is considered the first round, how many people will have been given fliers after 4 rounds?

SOLUTION:

Substitute 1 for a_1 , 4 for r , 4 for n in the Sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{1 - 1(4)^4}{1 - 4} \\ &= \frac{1 - 256}{-3} \\ &= \frac{-255}{-3} \\ &= 85 \end{aligned}$$

Therefore, 85 people will have been given flier after 4 rounds.